

Phys 430 - 1st HOMEWORK

1. Discuss the following concepts (just writing formulas is not enough, use words)

Ergodic principle

Ensemble

Distribution function

Microcanonical Distribution Function

Subsystem

2. Consider the example we started discussing in the lecture: Suppose you throw N dice. Let d_i be the value of the i^{th} dice, and let $d = \sum_{i=1}^N d_i$. Let $\rho_N(x)$ be the probability that $d = x$ for a system of N dice.

- (a) Show that

$$\rho_N(x) = \sum_{i=1}^6 \rho_1(i) \rho_{N-1}(x - i) \quad (1)$$

- (b) Calculate and sketch $\rho_3(x)$

- (c) Using a computer, draw the graph of $\frac{\rho_N(x)}{\rho_N^{\text{max}}}$ versus $\frac{x}{N}$, where ρ_N^{max} is the maximum value of $\rho_N(x)$, for $N = 3, 4, 20$ and see that the probability distribution gets narrower as N increases.

- (d) Estimate the relative fluctuation of d , i.e. $\frac{\Delta f}{\langle f \rangle}$ for $N = 3, 4$, and 20

3. Consider two different quantum systems: system A and system B . Both of the systems can be only in two different states. If the wave function of the whole system is given by

$$\Psi = a_1 \Psi_A^1 \Psi_B^1 + a_2 \Psi_A^1 \Psi_B^2 + a_3 \Psi_A^2 \Psi_B^2 \quad (2)$$

where $|a_1|^2 + |a_2|^2 + |a_3|^2 = 1$ and Ψ_X^i ($X = A, B, i = 1, 2$) is the wavefunction of the i^{th} state of the X system. What is the quantum statistical matrix of system A ? What is the quantum statistical matrix of system B ?